

**$B \rightarrow K_1(1270) (\rightarrow \rho K) \ell^+ \ell^-$  in LEET**S. RAI CHOUDHURY<sup>1\*</sup>, A. S. CORNELL<sup>2†</sup>, NAVEEN GAUR<sup>3‡</sup><sup>1</sup> *Center for Theoretical Physics, Jamia Millia University, Delhi, India.*<sup>2</sup> *Université de Lyon 1, Institut de Physique Nucléaire, Villeurbanne, France.*<sup>3</sup> *Theory Division, KEK, 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan.***Abstract**

Flavour Changing Neutral Current decays of the  $B$ -meson are a very useful tool for studying possible physics scenarios beyond the Standard Model (SM), where of the many FCNC modes radiative, purely leptonic and semi-leptonic decays of the  $B$ -meson are relatively clean tests. Within this context the BELLE collaboration has searched for the  $B \rightarrow K_1(1270)\gamma$  process and provided an upper bound on this decay. In this work we have used this upper bound in studying the angular correlations for the related semi-leptonic decay mode  $B \rightarrow K_1(1270)(\rightarrow \rho K)\ell^+\ell^-$ , where we have used the form factors that have already been estimated for the  $B \rightarrow K_1(1270)\gamma$  mode. Note that the additional form factors that are required were calculated using the Large Energy Effective Theory (LEET).

---

<sup>\*</sup>src@physics.du.ac.in<sup>†</sup>cornell@ipnl.in2p3.fr<sup>‡</sup>naveen@post.kek.jp

# 1 Introduction

The Flavour Changing Neutral Current (FCNC) decays of the  $B$ -meson are an important tool for investigating possible physics scenarios beyond the Standard Model (SM), where such decays are forbidden at tree level. It is for this reason that FCNC processes are very sensitive to possible small corrections that may be a result of any modification to the SM, or from some new interactions. Of the FCNC decays the radiative mode  $B \rightarrow K^* \gamma$  has been experimentally measured, with a lot of theoretical work also having gone into its study. A related decay,  $B \rightarrow K^* \ell^+ \ell^-$ , has also been observed experimentally. This latter process offers many more observables for confrontation with theory (such as Forward-Backward (FB) asymmetries, polarizations and angular correlations between the final state particles etc.), where the theoretical work for this subject has spawned many investigations. Recently the radiative mode  $B \rightarrow K_2^* \gamma$  has also been observed with good limits also being set on the modes  $B \rightarrow K_1(1270, 1400) \gamma$ , where the  $K_1$ 's are the  $1^+$  resonances. The numbers for these rates are comparable to those for the  $K^*(890)$  resonance case and we may expect that with more data becoming available the related  $B \rightarrow K_1 \ell^+ \ell^-$  would be observed just as with the  $K^*(890)$  case. Furthermore, as with the  $K^*(890)$ , such data will provide an independent opportunity to test the predictions of the SM.

In this paper we study the angular distribution of the rare  $B$ -decay  $B \rightarrow K_1(1270) (\rightarrow \rho K) \ell^+ \ell^-$ , which may be expected to be observed in future B-factories. We use the standard effective Hamiltonian approach, and use the form factors that have already been estimated for the corresponding radiative decay  $B \rightarrow K_1(1270) \gamma$ . The additional form factors for the dileptonic channel are estimated using the Large Energy Effective Theory (LEET), which enables one to relate the additional form factors to the form factors of the radiative mode. Our results provide, just like in the case of the  $K^*(890)$  resonance, an opportunity for a straightforward comparison of the basic theory with experimental results, which may be expected in the near future for this channel. Recall that the physical  $K_1(1270)$  and  $K_1(1400)$  states are the mixture of  $K_{1A}$  ( $^3P_1$  state) and  $K_{1B}$  ( $^1P_1$  state) states [3]:

$$\begin{aligned} K_1(1270) &= K_{1A} \sin\theta + K_{1B} \cos\theta , \\ K_1(1400) &= K_{1A} \cos\theta - K_{1B} \sin\theta , \end{aligned} \tag{1}$$

where  $\theta$  is the  $K_1(1270) - K_1(1400)$  mixing angle. Cheng *et al.* [3] proposed two possible solutions for this angle, namely  $\theta = \pm 37^\circ, \pm 58^\circ$ . Of these possibilities the negative values of the mixing angles predict the Branching Ratio (BR) of  $B \rightarrow K_1(1400) \gamma$  to be more than that of  $B \rightarrow K_1(1270) \gamma$ , which is disfavoured from experimental data (although not ruled out). For our work we have taken the positive values of the mixing angles.

The paper is organized as follows: In section II we will give the relevant effective Hamiltonian and the LEET form factors for the process under consideration. In section III we will give the expressions for the differential decay rate for the semi-leptonic decay mode under consideration. Finally we will conclude with our results in section IV.

## 2 The Effective Hamiltonian and form factors

The short distance contribution to the decay  $B \rightarrow K_1 \ell^+ \ell^-$  is governed by the quark level decay  $b \rightarrow s \ell^+ \ell^-$ , and where the SM operator basis can be described by the effective Hamiltonian:

$$\mathcal{H}^{eff} = \frac{G_F \alpha}{\sqrt{2} \pi} V_{ts}^* V_{tb} \left[ -2C_7^{eff} m_b \left( \bar{s}_R i \sigma_{\mu\nu} \frac{q^\nu}{q^2} b_R \right) (\bar{\ell} \gamma_\mu \ell) + C_9^{eff} (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \ell) + C_{10} (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \gamma_5 \ell) \right] . \quad (2)$$

We can rewrite the above effective Hamiltonian in the following form:

$$\begin{aligned} \mathcal{H}^{eff} &= \frac{G_F \alpha}{\sqrt{2} \pi} V_{ts}^* V_{tb} \left[ -2C_7^{eff} m_b \left( \bar{s}_R i \sigma_{\mu\nu} \frac{q^\nu}{q^2} b_R \right) (\bar{\ell} \gamma_\mu \ell) + (C_9^{eff} - C_{10}) (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \ell_L) \right. \\ &\quad \left. + (C_9^{eff} + C_{10}) (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \ell_R) \right] , \\ &= \frac{G_F \alpha}{\sqrt{2} \pi} V_{ts}^* V_{tb} \left[ -2iC_7^{eff} m_b \frac{q^\nu}{q^2} (T_{\mu\nu} + T_{\mu\nu}^5) (\bar{\ell} \gamma^\mu \ell) + (C_9^{eff} - C_{10}) (V - A)_\mu (\bar{\ell} \gamma^\mu \ell_L) \right. \\ &\quad \left. + (C_9^{eff} + C_{10}) (V - A)_\mu (\bar{\ell} \gamma^\mu \ell_R) \right] , \end{aligned} \quad (3)$$

with

$$V_\mu = \frac{1}{2} (\bar{s} \gamma_\mu b) , \quad (4)$$

$$A_\mu = \frac{1}{2} (\bar{s} \gamma_\mu \gamma_5 b) , \quad (5)$$

$$T_{\mu\nu} = \frac{1}{2} (\bar{s} \sigma_{\mu\nu} b) , \quad (6)$$

$$T_{\mu\nu}^5 = \frac{1}{2} (\bar{s} \sigma_{\mu\nu} \gamma_5 b) . \quad (7)$$

In equation (3) we have used the  $(V-A)$  structure for the hadronic part (except for  $C_7$ ). Note that this structure doesn't change under the transformation  $V \leftrightarrow -A$  and  $T_{\mu\nu} \leftrightarrow T_{\mu\nu}^5$ . Furthermore, we can relate the hadronic factors of  $T_{\mu\nu}$  and  $T_{\mu\nu}^5$  by using the identity<sup>§</sup>:

$$\sigma_{\mu\nu} = -\frac{i}{2} \varepsilon^{\mu\nu\rho\delta} \sigma_{\rho\delta} \gamma_5 .$$

In this work we shall closely follow the notation used by Kim *et al.*[1] by defining the form factors of  $K_1(1270)$  as:

$$\langle K_1(p') | \bar{s} \gamma_\mu b | B(p) \rangle = -f \epsilon_\mu^* - a_+ (\epsilon^* \cdot p) (p + p')_\mu - a_- (\epsilon^* \cdot p) (p - p')_\mu , \quad (8)$$

$$\langle K_1(p') | \bar{s} \gamma_\mu \gamma_5 b | B(p) \rangle = -ig \varepsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} (p + p')^\lambda (p - p')^\sigma$$

---

<sup>§</sup>Where we have used the convention that  $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$  and that  $\varepsilon_{0123} = 1$ .

$$= 2ig\varepsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu}p^\lambda(p')^\sigma , \quad (9)$$

$$\begin{aligned} \langle K_1(p')|\bar{s}\sigma_{\mu\nu}\gamma_5 b|B(p)\rangle &= g_+\varepsilon_{\mu\nu\lambda\sigma}\epsilon^{*\lambda}(p+p')^\sigma + g_-\varepsilon_{\mu\nu\lambda\sigma}\epsilon^{*\lambda}(p-p')^\sigma \\ &\quad + h\varepsilon_{\mu\nu\lambda\sigma}(p+p')^\lambda(p-p')^\sigma(\epsilon.p) , \end{aligned} \quad (10)$$

$$\begin{aligned} \langle K_1(p')|\bar{s}\sigma_{\mu\nu} b|B(p)\rangle &= -ig_+\left[\epsilon_\nu^*(p+p')_\mu - \epsilon_\mu^*(p+p')_\nu\right] - ig_-\left[\epsilon_\nu^*(p-p')_\mu - \epsilon_\mu^*(p-p')_\nu\right] \\ &\quad - i2h\left(p_\mu p'_\nu - p_\nu p'_\mu\right)(\epsilon^*.p) . \end{aligned} \quad (11)$$

From the above equations we can observe that there are seven form factors which govern the  $B \rightarrow K_1$  transition, where we will now relate these form factors using the LEET approach. Note that the advantage of writing the form factors in this form is that the expressions of the amplitudes ( $\mathcal{A}_R, \mathcal{A}_L$ ) are as given in equations (17) and (18) in Kim *et al.*[1], due to the symmetry of the expressions under the exchange  $V \leftrightarrow -A$ ,  $T_{\mu\nu} \rightarrow T_{\mu\nu}^5$ ,  $T_{\mu\nu}^5 \rightarrow T_{\mu\nu}$ .

Note also that Cheng and Chua have parameterized the tensorial form factors for the  $B \rightarrow K_1$  transition as [3]:

$$\begin{aligned} \langle K_{1A,1B}(p')|\bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p)\rangle &= i\varepsilon_{\mu\nu\lambda\rho}\epsilon^{*\nu}P^\lambda q^\rho Y_{A1,B1} + (\epsilon_\mu^*P.q - P_\mu\epsilon^*.q)Y_{A2,B2} \\ &\quad + \epsilon^*.q\left[q_\mu - P_\mu\frac{q^2}{P.q}\right]Y_{A3,B3} , \end{aligned} \quad (12)$$

where  $P = p + p'$  and  $q = p - p'$ . The  $K_{1A}$  and  $K_{1B}$  states are the angular momentum eigenstates as defined in equation (1). Using equation (1) we can define the physical  $B \rightarrow K_1(1270)$  form factors as:

$$Y_i^{B \rightarrow K_1(1270)} = Y_{Ai}(q^2)\sin\theta + Y_{Bi}(q^2)\cos\theta , \quad i = 1, 2, 3 . \quad (13)$$

The parameterizations of the form-factors  $Y_i$ , as used by Cheng and Chua, are given in Appendix A. These form factors can be related to the tensorial form factors given in equations (9)-(12) by:

$$Y_1^{B \rightarrow K_1(1270)} = -g_+ , \quad (14)$$

$$Y_2^{B \rightarrow K_1(1270)} = -g_+ - g_-\frac{q^2}{P.q} , \quad (15)$$

$$Y_3^{B \rightarrow K_1(1270)} = g_- + h(P.q) . \quad (16)$$

Using the LEET approach as given by Charles *et al.*[2] we can obtain the following relations between the form factors:

$$f = 2MEg , \quad (17)$$

$$g_+ = -gM , \quad (18)$$

$$g_- = gM , \quad (19)$$

$$a_+ = -a_- = -(g + hM) , \quad (20)$$

where  $M$  is the mass of the parent hadron and  $E$  is the energy of the daughter hadron. We now define all the form factors in terms of just two independent form factors ( $g$  and  $h$ ).

Using the LEET relations as given in equations (17)-(20) and equations (13)-(16), the form factors for the  $B \rightarrow K_1$  transition can be related to Cheng and Chua's form factors as<sup>¶</sup>:

$$\begin{aligned}
g_+ &= -Y_1^{B \rightarrow K_1(1270)} , \\
g_- &= Y_1^{B \rightarrow K_1(1270)} , \\
g &= \frac{Y_1^{B \rightarrow K_1(1270)}}{M} , \\
f &= 2EY_1^{B \rightarrow K_1(1270)} , \\
h &= \frac{Y_3^{B \rightarrow K_1(1270)} - Y_1^{B \rightarrow K_1(1270)}}{M^2 - m_V^2} , \\
a_+ &= -a_- = \frac{Y_1^{B \rightarrow K_1(1270)}m_V^2 - Y_3^{B \rightarrow K_1(1270)}M^2}{M(M^2 - m_V^2)} , 
\end{aligned} \tag{21}$$

where  $M$  is the mass of the  $B$ -meson and  $m_V$  is the mass of the  $K_1$ .

### 3 Kinematics and differential decay rate

In the following it is convenient to define our kinematics in terms of the following vectors:

$$P = p' = p_\rho + p_K , \quad Q = p_\rho - p_K , \quad L = p_+ + p_- , \quad N = p_+ - p_- .$$

Subsequently the decay mode  $K_1 \rightarrow \rho K$ , of the  $K_1$  meson, can be parameterized by the matrix element [4]:

$$\begin{aligned}
\mathcal{M}(K_1(p') \rightarrow \rho(p_\rho)K(p_K)) &= \frac{2g_{K_1\rho K}}{m_{K_1}m_\rho} \left[ (p' \cdot p_\rho)(\epsilon_\rho \cdot \epsilon_{K_1}) - (p' \cdot \epsilon_\rho)(p_\rho \cdot \epsilon_{K_1}) \right] \\
&= \frac{g_{K_1\rho K}}{m_{K_1}m_\rho} \left[ (P^2 + P \cdot Q)(\epsilon_\rho \cdot \epsilon_{K_1}) - (P \cdot \epsilon_\rho)(Q \cdot \epsilon_{K_1}) \right] . 
\end{aligned} \tag{22}$$

This matrix element will give the decay width [4]:

$$\Gamma_{K_1} = \frac{|g_{K_1\rho K}|^2}{2\pi m_{K_1}^2} q' \left( 1 + \frac{2}{3} \frac{q'^2}{m_\rho^2} \right) , \tag{23}$$

with  $q' = \frac{1}{2m_{K_1}} \lambda^{1/2}(m_{K_1}^2, m_\rho^2, m_K^2)$ , and where  $p'$ ,  $\epsilon_{K_1}$  and  $p_\rho$ ,  $\epsilon_\rho$  are the momentum and polarization vectors of  $K_1$  and  $\rho$  respectively. In the following analysis we shall neglect the masses of the leptons, the kaon and the  $\rho$ , where in the above we have used  $p' = P$  and  $p_\rho = (P + Q)/2$ .

The final 4-body decay amplitude can be written as the sum of two amplitudes:

$$\mathcal{A} = \left[ \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha m_b}{\pi L^2} \left( \frac{g_{K_1\rho K}}{m_{K_1}m_\rho} \right) \right] (\epsilon_\rho)_\beta (\mathcal{A}_R^\beta + \mathcal{A}_L^\beta) , \tag{24}$$

---

<sup>¶</sup>It is important to note that the notations of the Levi-Civita tensor in Cheng and Chua's paper [3] differs from the notation of Kim *et al.* [1] by a overall negative sign. We are following the notation of Kim *et al.*

where

$$\begin{aligned} \mathcal{A}_R^\beta &= (\bar{\ell}_R \gamma^\mu l_R) \left( a_R g_{\mu\nu} - b_R P_\mu L_\nu + i c_R \epsilon_{\mu\nu\alpha\beta} P^\alpha L^\beta \right) \frac{g^{\nu\alpha} - P^\nu P^\alpha / m_{K_1}^2}{P^2 - m_{K_1}^2 + i m_{K_1} \Gamma_{K_1}} \\ &\quad \times \left[ (P^2 + P \cdot Q) g_{\alpha\beta} - P_\beta Q_\alpha \right] , \end{aligned} \quad (25)$$

$$\begin{aligned} \mathcal{A}_L^\beta &= (\bar{\ell}_L \gamma^\mu l_L) \left( a_L g_{\mu\nu} - b_L P_\mu L_\nu + i c_L \epsilon_{\mu\nu\alpha\beta} P^\alpha L^\beta \right) \frac{g^{\nu\alpha} - P^\nu P^\alpha / m_{K_1}^2}{P^2 - m_{K_1}^2 + i m_{K_1} \Gamma_{K_1}} \\ &\quad \times \left[ (P^2 + P \cdot Q) g_{\alpha\beta} - P_\beta Q_\alpha \right] . \end{aligned} \quad (26)$$

The  $a_R$ ,  $b_R$ ,  $c_R$  and  $a_L$ ,  $b_L$ ,  $c_L$  can be expressed as:

$$a_L = -C_7 \left[ 2(P \cdot L)g_+ + L^2(g_+ + g_-) \right] + \frac{(C_9^{eff} - C_{10})f}{2m_b} L^2 , \quad (27)$$

$$b_L = -2C_7(g_+ - L^2 h) - \frac{(C_9^{eff} - C_{10})a_+}{m_b} L^2 , \quad (28)$$

$$c_L = -2C_7 g_+ - \frac{(C_9^{eff} - C_{10})g}{m_b} L^2 , \quad (29)$$

$$a_R = -C_7 \left[ 2(P \cdot L)g_+ + L^2(g_+ + g_-) \right] + \frac{(C_9^{eff} + C_{10})f}{2m_b} L^2 , \quad (30)$$

$$b_R = -2C_7(g_+ - L^2 h) - \frac{(C_9^{eff} + C_{10})a_+}{m_b} L^2 , \quad (31)$$

$$c_R = -2C_7 g_+ - \frac{(C_9^{eff} + C_{10})g}{m_b} L^2 . \quad (32)$$

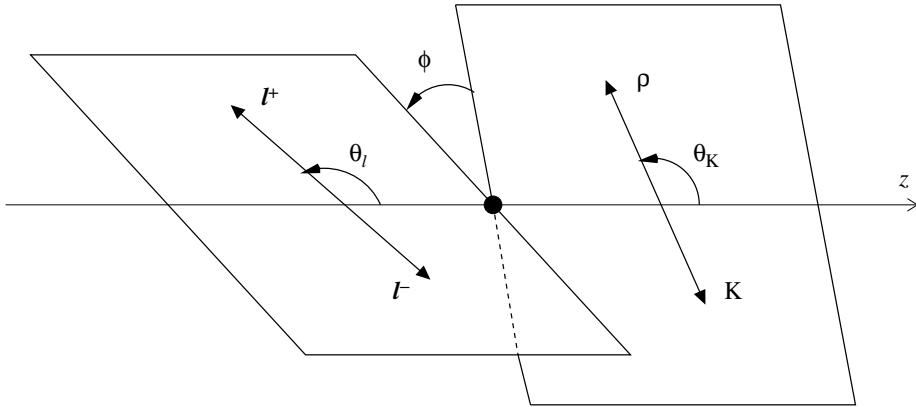


Figure 1: The definition of the kinematical variables in the decay  $B \rightarrow K_1(\rightarrow \rho K)\ell^+\ell^-$ .

As such the decay rate can be computed, with the result:

$$\frac{d^5\Gamma}{dp^2 dl^2 d\cos\theta_K d\cos\theta_+ d\phi} = \frac{2\sqrt{\lambda}}{128 \times 256\pi^6 m_B^3} \left| \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha m_b}{\pi L^2} \left( \frac{g_{K_1\rho K}}{m_{K_1} m_\rho} \right) \right|^2 (|A_R|^2 + |A_L|^2) \quad (33)$$

where

$$|A_{\{L,R\}}|^2 = \left( -g_{\alpha\beta} + \frac{(p_\rho)_\alpha (p_\rho)_\beta}{m_\rho^2} \right) (\mathcal{A}_{\{L,R\}})^\alpha (\mathcal{A}_{\{L,R\}}^*)^\beta, \quad (34)$$

where the various angles used above are as shown in figure 1. Recall that we shall present our results in terms of the vectors  $P$ ,  $Q$ ,  $L$  and  $N$ , by use of the transformation:

$$p_\rho \rightarrow \frac{P+Q}{2}, \quad p_K \rightarrow \frac{P-Q}{2}, \quad p_+ \rightarrow \frac{L+N}{2}, \quad p_- \rightarrow \frac{L-N}{2}.$$

Using the kinematics as prescribed in Kim *et al.*[1], that is, where we set  $p = \sqrt{p_{K_1}^2}$ ,  $l = \sqrt{(p_+ + p_-)^2}$  and  $\lambda = \frac{1}{4}(m_B^2 - p^2 - l^2)^2 - p^2 l^2$ . Furthermore, we shall introduce various angles, namely  $\theta_K$  as the polar angle of the  $\rho$  momentum in the rest frame of the  $K_1$  meson with respect to the helicity axis, *i.e.* the outgoing direction of  $K_1$ . Similarly  $\theta_+$  as the polar angle of the positron in the dilepton CM frame with respect to the  $K_1$  momentum. And finally  $\phi$  as the azimuthal angle between these planes, that is, the  $K_1 \rightarrow \rho K$  and  $B \rightarrow K_1 \ell^+ \ell^-$  planes. In this case:

$$\begin{aligned} |A_L|^2 = & \frac{1}{(P^2 - m_{K_1}^2)^2 + (m_{K_1} \Gamma_{K_1})^2} \frac{1}{2} \left[ |a_L|^2 \left\{ 2(P^2 + 2P \cdot Q) \left( (L \cdot P)^2 - (N \cdot P)^2 \right) + 2((N \cdot Q)^2 \right. \right. \\ & - (L \cdot Q)^2 + L^2(P \cdot Q) - N^2(P \cdot Q)) P^2 + 4((L \cdot Q)(L \cdot P) + (N \cdot P)(N \cdot Q))(P \cdot Q) + (L^2 - N^2) \\ & \times (P^2 + Q^2) P^2 \left. \right\} + |b_L|^2 \left\{ \left( 2(L \cdot P)^2 - 2(N \cdot P)^2 + (N^2 - L^2) P^2 \right) \left( (P^2 + 2(P \cdot Q))(L \cdot P)^2 \right. \right. \\ & - (P^2 + (P \cdot Q))^2 L^2 + 2(L \cdot Q)(P \cdot Q)(L \cdot P) - (L \cdot Q)^2 P^2 \left. \right\} + |c_L|^2 \left\{ \left[ N^2 Q^2 P^2 - 2(N \cdot Q)^2 P^2 \right. \right. \\ & + \left( 2P^4 + (4(P \cdot Q) + Q^2) P^2 + 2(P \cdot Q)^2 \right) P^2 + 2(P \cdot Q)^2 \left. \right] (L \cdot P)^2 + 4(N \cdot P)(N \cdot Q)(L \cdot Q) \\ & \times (L \cdot P) P^2 - 2(N \cdot P)^2 (L \cdot Q)^2 P^2 - \left[ -2 \left( P^4 + (2(P \cdot Q) + Q^2) P^2 + (P \cdot Q)^2 \right) (N \cdot P)^2 \right. \\ & + 4(N \cdot Q)(P \cdot Q)(N \cdot P) P^2 + \left( -2P^2(N \cdot Q)^2 + N^2(P^2 Q^2 - (P \cdot Q)^2) + L^2(2P^4 + (4(P \cdot Q) \right. \\ & + Q^2) P^2 + (P \cdot Q)^2) \left. \right] P^2 \left. \right\} + 4Re(a_L b_L^*) \left\{ - (P^2 + 2(P \cdot Q))(L \cdot P)^3 - 2(L \cdot Q)(P \cdot Q)(L \cdot P)^2 \right. \\ & + \left( P^2 (L \cdot Q)^2 + (P^2 + (P \cdot Q))^2 L^2 + (N \cdot P)(N \cdot Q)(P \cdot Q) + (N \cdot P)^2 (P^2 + 2(P \cdot Q)) \right) (L \cdot P) \\ & + (L \cdot Q)(N \cdot P) \left( (N \cdot P)(P \cdot Q) - (N \cdot Q) P^2 \right) \left. \right\} + 4Re(a_L c_L) \left\{ (N \cdot Q)(P \cdot Q)(L \cdot P)^2 + (N \cdot P) \right. \\ & \times (L \cdot Q)^2 P^2 - \left( (N \cdot Q) P^2 + (N \cdot P)(P \cdot Q) \right) (L \cdot Q)(L \cdot P) \left. \right\} - 4Re(b_L c_L^*) \left\{ \left( (L \cdot Q) P^2 \right. \right. \\ & - (L \cdot P)(P \cdot Q) \left. \right) \left( - (N \cdot Q)(L \cdot P)^2 + (L \cdot Q)(N \cdot P)(L \cdot P) + \left[ (N \cdot Q) P^2 - (N \cdot P)(P \cdot Q) \right] L^2 \right) \left. \right\} \\ & + 4(L \widetilde{N} P Q) \left( (L \cdot P)(P \cdot Q) - (L \cdot Q) P^2 \right) \left( Im(a_L b_L^*) + (N \cdot P) Im(b_L c_L^*) \right) \end{aligned}$$

$$+4Im(a_L c_L^*)(L\widetilde{N}PQ)\left((N.Q)P^2 - (N.P)(P.Q)\right)\Bigg] , \quad (35)$$

$$\begin{aligned}
|A_R|^2 = & \frac{1}{(P^2 - m_{K_1}^2)^2 + (m_{K_1}\Gamma_{K_1})^2} \frac{1}{2} \Bigg[ |a_R|^2 \left\{ 2(P^2 + 2P.Q) \left( (L.P)^2 - (N.P)^2 \right) + 2((N.Q)^2 \right. \\
& - (L.Q)^2 + L^2(P.Q) - N^2(P.Q))P^2 + 4((L.Q)(L.P) + (N.P)(N.Q))(P.Q) + (L^2 - N^2) \\
& \times (P^2 + Q^2)P^2 \Big\} + |b_R|^2 \left\{ \left( 2(L.P)^2 - 2(N.P)^2 + (N^2 - L^2)P^2 \right) \left( (P^2 + 2(P.Q))(L.P)^2 \right. \right. \\
& - (P^2 + (P.Q))^2 L^2 + 2(L.Q)(P.Q)(L.P) - (L.Q)^2 P^2 \Big\} + |c_R|^2 \left\{ \left[ N^2 Q^2 P^2 - 2(N.Q)^2 P^2 \right. \right. \\
& + \left( 2P^4 + (4(P.Q) + Q^2)P^2 + 2(P.Q)^2 \right) P^2 + 2(P.Q)^2 \Big] (L.P)^2 + 4(N.P)(N.Q)(L.Q) \\
& \times (L.P)P^2 - 2(N.P)^2(L.Q)^2 P^2 - \left[ -2(P^4 + (2(P.Q) + Q^2)P^2 + (P.Q)^2)(N.P)^2 \right. \\
& + 4(N.Q)(P.Q)(N.P)P^2 + \left( -2P^2(N.Q)^2 + N^2(P^2 Q^2 - (P.Q)^2) + L^2(2P^4 + (4(P.Q) \right. \\
& + Q^2)P^2 + (P.Q)^2) \Big] P^2 \Big\} + 4Re(a_R b_R^*) \left\{ - (P^2 + 2(P.Q))(L.P)^3 - 2(L.Q)(P.Q)(L.P)^2 \right. \\
& + \left( P^2(L.Q)^2 + (P^2 + (P.Q))^2 L^2 + (N.P)(N.Q)(P.Q) + (N.P)^2(P^2 + 2(P.Q)) \right) (L.P) \\
& + (L.Q)(N.P) \left( (N.P)(P.Q) - (N.Q)P^2 \right) \Big\} - 4Re(a_R c_R) \left\{ (N.Q)(P.Q)(L.P)^2 + (N.P) \right. \\
& \times (L.Q)^2 P^2 - \left( (N.Q)P^2 + (N.P)(P.Q) \right) (L.Q)(L.P) \Big\} + 4Re(b_R c_R^*) \left\{ \left( (L.Q)P^2 \right. \right. \\
& - (L.P)(P.Q) \Big) \left( - (N.Q)(L.P)^2 + (L.Q)(N.P)(L.P) + \left[ (N.Q)P^2 - (N.P)(P.Q) \right] L^2 \right) \Big\} \\
& + 4(L\widetilde{N}PQ) \left( (L.P)(P.Q) - (L.Q)P^2 \right) \left( -Im(a_R b_R^*) + (N.P)Im(b_R c_R^*) \right) \\
& \left. + 4Im(a_R c_R^*)(L\widetilde{N}PQ) \left( (N.Q)P^2 - (N.P)(P.Q) \right) \right] , \quad (36)
\end{aligned}$$

where  $(A\widetilde{B}CD) = \varepsilon_{\alpha\beta\gamma\delta} A^\alpha B^\beta C^\gamma D^\delta$ .

The  $p^2$  integration is performed using the narrow width approximation of the  $K_1$  decay, *i.e.*:

$$\lim_{\Gamma_{K_1} \rightarrow 0} \frac{m_{K_1}\Gamma_{K_1}}{(P^2 - m_{K_1}^2)^2 + (m_{K_1}\Gamma_{K_1})^2} = \pi\delta(P^2 - m_{K_1}^2) . \quad (37)$$

As such, the total decay width can be expressed as:

$$\begin{aligned}
\Gamma = & \int p^2 \delta(p^2 - m_{K_1}^2) \int_{4m_\ell^2}^{(m_B - m_{K_1})^2} l^2 \int_{-1}^1 d(\cos\theta_K) \int_{-1}^1 d(\cos\theta_+) \int_0^{2\pi} d\phi \frac{2\sqrt{\lambda}}{128 \times 256\pi^6 m_B^3} \\
& \times \left| \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha m_b}{\pi L^2} \right|^2 \frac{2\pi^2}{m_{K_1} m_\rho^2 q' \left( 1 + \frac{2}{3} \frac{q'^2}{m_\rho^2} \right)} (|A_R|^2 + |A_L|^2) , \quad (38)
\end{aligned}$$



with  $q' = \frac{1}{2m_{K_1}}\lambda^{1/2}(m_{K_1}^2, m_\rho^2, m_K^2)$  and  $\lambda = \frac{1}{4}(m_B^2 - p^2 - l^2)^2 - p^2 l^2$ .

## 4 Results

The form factors for the radiative mode  $B \rightarrow K_1(1270)\gamma$ , as given by Cheng and Chua [3], assumed that the physical states  $K_1(1270)$  and  $K_1(1400)$  were mixtures of the angular momentum eigenstates  $K_{1A}$  and  $K_{1B}$ , where the mixing angle between these states is not known precisely (though it is believed to be such as to cause the maximal mixing between the states). Where the hypothesis of mixing between the two states naturally explains the suppression of one of the decay modes with respect to the other. In reference [3] the mixing angles suggested were  $\theta = \pm 37^\circ, \pm 58^\circ$ . The negative values of the mixing angles suggest the suppression of  $B \rightarrow K_1(1270)\gamma$  as compared to  $B \rightarrow K_1(1400)\gamma$ , which is disfavoured (although not conclusively ruled out) from the observation of the radiative decay mode  $B \rightarrow K_1(1270, 1400)\gamma$  by the BELLE collaboration [5].

Although the prescription of mixing between the states helps to explain the suppression of one of the modes, as compared to the other, the form factors as given in reference [3] predict a lower value of the branching ratio for  $B \rightarrow K_1(1270)\gamma$  as compared to experimental results. Note that there have been many attempts in references [6] to address this issue, where these attempts essentially predict a much larger value of the zero recoil value of the form factors. For our analysis we have used the form factors as given by Cheng and Chua [3]. Our analytical results for the LEET form factors and the differential decay rate retains the same form for any possible increase in the zero recoil value of the form factors. For our analysis we have used the mixing angle between the two angular momentum eigenstates ( $K_{1A}, K_{1B}$ ) to be ( $\theta =$ )  $58^\circ$ . Our SM value of the branching ratio for  $B \rightarrow K_1(\rightarrow \rho K)\ell^+\ell^-$ , using the input parameters as defined in Appendix B, is  $2.3 \times 10^{-7}$ .

In figure 2 we have shown the variation of the differential branching ratio of  $B \rightarrow K_1(\rightarrow \rho K)\ell^+\ell^-$  as a function of the dileptonic invariant mass. The two different lines in the plot correspond to the results of including and excluding the long distance charm resonances, where these long-distance effects can be included with the redefinition of  $C_9^{eff}$ . For this purpose we have used the prescription as given in Kruger and Sehgal [7]. Note that use of the LEET for obtaining the vector and axial vector form factors is justified more for relatively large values of  $s$ , where at the moment there are no first principle determinations of these form factors similar to the ones determined for the tensor form factors by Cheng and Chua [3]. We therefore continue using the LEET based form factors for the entire range. As and when more accurate values are available for the low  $s$ -region our expressions can easily be reevaluated for a fresh plot.

In figure 3 we have shown the plot of the differential branching ratio as a function of the azimuthal angle between the planes  $\rho K$  and  $\ell^+\ell^-$ . And in our final plot we have shown the dependence of the differential branching ratio as a function of the scattering angle  $\theta_\ell$  in  $\ell^+\ell^-$ , as defined in figure 1.

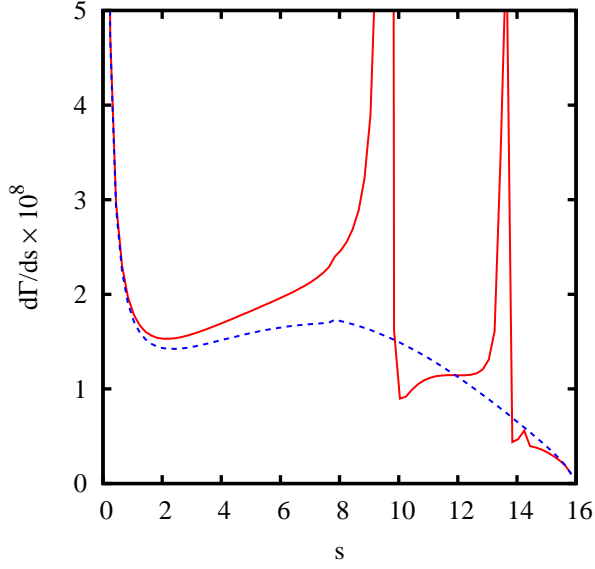


Figure 2: *The differential branching ratio as a function of dilepton invariant mass. The two lines correspond to the result of switching on (solid line) and off (dashed line) the charmonium resonances in  $C_9^{eff}$ .*

We should like to note, at this point, that for any new physics model that can be effectively absorbed by the “standard” set of Wilson coefficients in the effective Hamiltonian, our analytic results given in previous sections can be used to obtain the corresponding change in the angular correlations.

As one final remark we may integrate our differential decay rate over the final state hadrons to get the decay rate of the process  $B \rightarrow K_1 \ell^+ \ell^-$ . Note that by using our definition of the form factors we can relate these to the ones for  $K^*(890)$ , under the substitutions  $V \leftrightarrow -A$  and  $T \leftrightarrow T_5$ , such that the corresponding decay rates are obtained easily (by means of this substitution). It is easily seen then that the location of the zero in the FB asymmetry of this integrated decay rate is the same as in the  $K^*(890)$  case, with the numerical value of the form factors being different. However, though the zeroes can be related, the overall shape of the FB asymmetries could be different from  $B \rightarrow K^*(890)$ .

To conclude, the mode which we have studied above, within the SM level, can in principal be measured at present  $B$ -factories. Various angular correlations of this mode can also be studied in future SuperB factories. The study of the various angular correlations in  $B \rightarrow K_1(\rightarrow \rho K) \ell^+ \ell^-$  can provide us with a very useful cross-checking tool for the SM and possible new physics in  $b \rightarrow s \ell^+ \ell^-$  transitions. In this pursuit we have given the LEET form factors for  $B \rightarrow K_1(1270) \ell^+ \ell^-$ , which could be useful in not only testing the operator structure of the SM but also the existence of possible new physics beyond it.

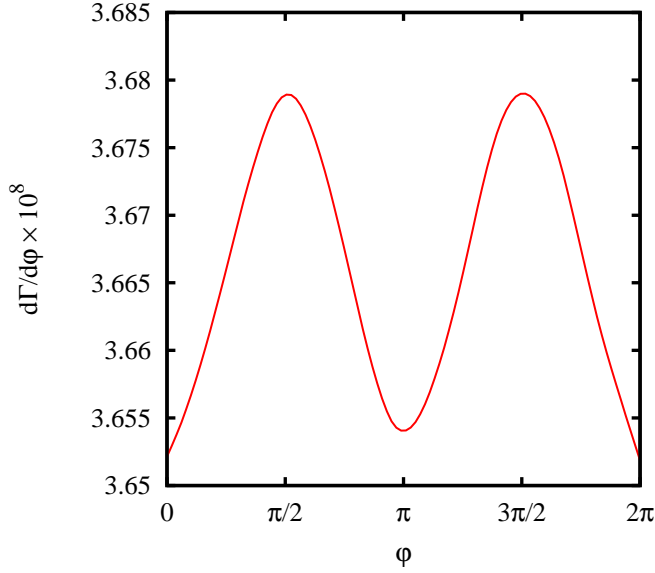


Figure 3: *The differential branching ratio as a function of the azimuthal angle  $\phi$ .*

## Acknowledgments

The work of SRC is supported by the Department of Science & Technology (DST), India. SRC would also like to thank the Theory Division, KEK, for their local hospitality, where the part of this work was done. The work of NG is supported by JSPS under the grant no. P06043.

## A The form factors

Firstly, it is important to note that the additional negative sign in equation (19) is due to the difference in our definition of  $\varepsilon$ , as compared to that of Cheng and Chua [3], Charles *et al.*[2] and Kim *et al.*[1]. Furthermore, note that the definitions of Cheng *et al.* are the same as that of Charles *et al.*, but differ from that of Kim *et al.* by a sign, which can be taken into account by changing the sign in equation (19).

The form factors, as defined in Cheng *et al.* (for all the form factors except  $Y_{B3}$ )[3], are:

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2} . \quad (39)$$

For  $Y_{B3}$  we use [3]:

$$F(q^2) = \frac{F(0)}{(1 - q^2/m_B^2)(1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2)} . \quad (40)$$

The numerical values of the factors appearing in equation (22) and equation (23) are given in Table 1.

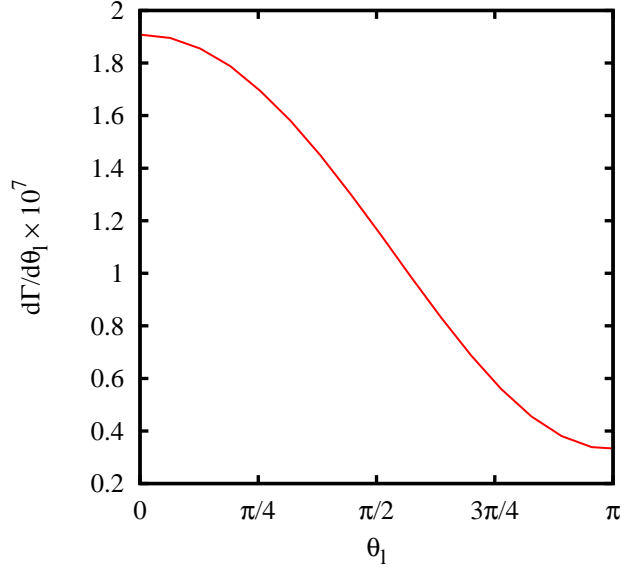


Figure 4: *The differential branching ratio as a function of  $\theta_\ell$ .*

F	F(0)	a	b
$Y_{A1}$	0.11	0.68	0.35
$Y_{A3}$	0.19	1.02	0.35
$Y_{B1}$	0.13	1.94	1.53
$Y_{B3}$	- 0.07	1.93	2.33

Table 1: *The form factors for the  $Y_{A_i}$  and  $Y_{B_i}$  [3].*

## B Input Parameters

$$\begin{aligned}
m_B &= 5.26 \text{ GeV}, & m_{K_1} &= 1.27 \text{ GeV}, & m_b &= 4.8 \text{ GeV}, & m_\rho &= 0.77 \text{ GeV}, & m_K &= 0.134 \text{ GeV}, \\
\alpha &= \frac{1}{129}, & G_F &= 1.17 \times 10^{-5} \text{ GeV}^{-2}, & |V_{tb}V_{ts}^*| &= 0.0385, \\
C_7 &= -0.3, & \text{and } C_{10} &= -4.6,
\end{aligned}$$

## References

- [1] C. S. Kim, Y. G. Kim, C. D. Lu and T. Morozumi, Phys. Rev. D **62**, 034013 (2000) [arXiv:hep-ph/0001151].
- [2] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D **60**, 014001 (1999) [arXiv:hep-ph/9812358].
- [3] H. Y. Cheng and C. K. Chua, Phys. Rev. D **69**, 094007 (2004) [arXiv:hep-ph/0401141].

- [4] L. Roca, J. E. Palomar and E. Oset, Phys. Rev. D **70**, 094006 (2004) [arXiv:hep-ph/0306188].
- [5] H. Yang *et al.*, Phys. Rev. Lett. **94**, 111802 (2005) [arXiv:hep-ex/0412039].
- [6] Y. J. Kwon and J. P. Lee, Phys. Rev. D **71**, 014009 (2005) [arXiv:hep-ph/0409133] ; G. Nardulli and T. N. Pham, Phys. Lett. B **623**, 65 (2005) [arXiv:hep-ph/0505048] ; J. P. Lee, Phys. Rev. D **74**, 074001 (2006) [arXiv:hep-ph/0608087].
- [7] F. Kruger and L. M. Sehgal, Phys. Lett. B **380**, 199 (1996) [arXiv:hep-ph/9603237].